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Determination of Macroscopic Electro-Mechanical Characteristics of 1-3 Piezoceramic/Polymer Composites by a Concentric Tube Model

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Abstract—An axisymmetric concentric tube model of a piezoelectric rod and a concentric elastic tube is used to characterize 1-3 piezoelectric/elastic composites macroscopically. The model is based on the following assumptions: the wavelength of mechanical waves is large compared to the distance between adjacent rods, displacements follow the separable static solutions for tubes, the electric field is constant, and displacements are continuous across the interface between the rod and the tube. With average displacements of and total forces on the surfaces as the mechanical degrees of freedom, and with charge and potential at the ends of the rod as the electrical degrees of freedom, the relation between the electromechanical degrees of freedom is given in matrix formulation. The formulation resembles the use of element stiffness matrices in the finite element method. A recursive numerical scheme for combining the matrix for the tube and the piezoelectric rod into one for the composite piezoelectric rod is used to directly identify the majority of the e -set of constitutive constants for 1-3 composites. The remaining constitutive constants are estimated from a cubes model, to allow inversion of the d - g , and h -sets and calculation of other characteristics.

I. INTRODUCTION

PIEZOELECTRIC/POLYMER COMPOSITES have promising characteristics for use in ultrasonic transducers. Due to the low transverse coupling factor, k_t , they can be used for undiced array transducers, but also as replacement in more conventional transducers, they offer a higher longitudinal coupling factor, k_{33} , lower acoustic impedance and higher damping than pure piezoceramics do. In a strict sense they also have a higher degree of anisotropy, stop-bands, and other characteristics that greatly complicate an analysis. For practical use, however, these effects are often negligible. Furthermore there is considerable variation in the characteristics of the constituent materials and some randomness in the structure of the composite, which preclude very accurate estimates. The aim of the present paper is to present a simple, and

yet fairly accurate model, the concentric tube model, which can be used to characterize 1-3 composites by average values, as if they were homogeneous piezoceramics. The concentric tube model estimates the six technically most important constants of the nine constants in the e -set.

The classical models [1] are the Voigt model, which assumes constant strain in the material, and the Reuss model, which assumes constant stress. For a 1-3 composite, i.e., piezoelectric rods in an elastic matrix, Young's modulus may be found with excellent agreement using the Voigt model [2], and the Bulk modulus of a 1-3 composite can be well estimated using the Reuss model [3]. For other connectivities or constants these models are not appropriate. More refined models are: The parallel-serial model [4], the cube model [5], and the method by Chan and Unsworth [6]. These methods are all based on the assumptions that either a strain component or the corresponding stress component is constant in the two phases, and they lead to selective use of the mixture rules of the Voigt or the Reuss type, possibly to repeated use of such rules. Hashimoto and Yamaguchi solves the one-dimensional problem of a 2-2 composite as well [7].

Unlike the methods in [1]–[7], the concentric tube model introduces explicit displacement assumption that leads to inhomogeneous strains, that are physically realistic, by choosing the quasi-static solutions as displacement functions. The essence of the method is to model a composite rod under axisymmetric load, and then to assume the composite rod to be typical for a larger volume of a 1-3 composite. This assumption is supported by the following observation. Consider a composite of circular rods in a hexagonal pattern. Under uniaxial stress in the axial direction, points with zero transverse displacement will form a hexagonal pattern. The largest distance between one of the hexagons and an average circle, is as little as 0.067 times the radius of the circle. Neglecting this difference is far from equivalent to introducing an error of the same magnitude, but is rather an averaging at an early stage. A consequence of the axisymmetry of the concentric tube model is that it leads to transversely isotropic material data which is true for the circular rods in a hexagonal pattern, but only approximately true for less symmetric structures.

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II. CONCENTRIC TUBE MODEL OF 1-3 COMPOSITES

The building block for the concentric tube model is the circular piezoelectric tube shown in Fig. 1. The displacement, u , and the electric potential, ϕ , are assumed to follow the separable axisymmetric quasi-static solutions:

$$u_r = a_r x_r + \frac{a_i}{x_r}; \quad u_z = a_z x_z; \quad \phi = a_\phi x_z \quad (1)$$

$$\begin{pmatrix} F_z \\ F_r \\ F_R \end{pmatrix} = 2\pi \begin{pmatrix} C_{33} \frac{R^2 - r^2}{2h} & -C_{13}r & C_{13}R \\ -C_{13}r & C_{11}h \frac{R^2 + r^2}{R^2 - r^2} - C_{12}h & -C_{11}h \frac{2Rr}{R^2 - r^2} \\ C_{13}R & -C_{11}h \frac{2Rr}{R^2 - r^2} & C_{11}h \frac{R^2 + r^2}{R^2 - r^2} + C_{12}h \end{pmatrix} \begin{pmatrix} U_z \\ U_r \\ U_R \end{pmatrix} \quad (4)$$

x_r and x_z are the coordinates in the radial and axial direction respectively. The indices on u refer to the direction of the displacement. a_r , a_i , a_z , and a_ϕ are constants, that are determined from the voltage, V , across the tube and the following typical (nodal) displacements that are shown in Fig. 1: U_r and U_R are the radial displacement at the inner and the outer surface of the tube, and U_z is the axial

where h is the height of the rod, r is the radius of the rod that equals the inner radius of the tube; F_z and F_r are the forces on the plane and the curved face of the rod and Q is the electric charge. The elastic tube gives

$$2\pi \begin{bmatrix} C_{33}^E \frac{r^2}{2h} + C_{33} \frac{R^2 - r^2}{2h} & -e_{33} \frac{r^2}{2h} & (C_{13}^E - C_{13})r & C_{13}R \\ -e_{33} \frac{r^2}{2h} & -\epsilon_{33}^S \frac{r^2}{2h} & -e_{31}r & 0 \\ (C_{13}^E - C_{13})r & -e_{31}r & (C_{11}^E + C_{12}^E - C_{12})h + C_{11}h \frac{R^2 + r^2}{R^2 - r^2} & -C_{11}h \frac{2Rr}{R^2 - r^2} \\ C_{13}^E R & 0 & -C_{11}h \frac{2Rr}{R^2 - r^2} & C_{11}h \frac{R^2 + r^2}{R^2 - r^2} + C_{12}h \end{bmatrix} \begin{pmatrix} U_z \\ U_r \\ U_R \end{pmatrix} \quad (5)$$

displacement difference between the two ends of the tube. The displacements and the electric potential are used to derive the strain vector, S_j , and the electric field vector, E_β . The stress vector, T_i , and the dielectric displacement vector, D_α , are then found, using the e -set of the constitutive equations [8]:

$$\begin{aligned} T_i &= C_{ij}^E S_j - e_{\beta i} E_\beta \\ D_\alpha &= e_{\alpha j} S_j + \epsilon_{\alpha\beta}^S E_\beta \end{aligned} \quad (2)$$

where $\alpha, \beta = 1, 2, 3$; $i, j = 1, \dots, 6$; and summation over repeated indices is implied. Integration of normal stress and dielectric displacement over the surfaces of the tube finally gives the total forces and total electric charge.

For the actual model only two special cases of the piezoelectric tube are needed, the piezoelectric rod and the elastic tube. The "stiffness" matrix for the piezoelectric rod can be written as

$$\begin{pmatrix} F_z \\ -Q \\ F_r \end{pmatrix} = 2\pi \begin{pmatrix} C_{33}^E \frac{r^2}{2h} & -e_{33} \frac{r^2}{2h} & C_{13}^E r \\ -e_{33} \frac{r^2}{2h} & -\epsilon_{33}^S \frac{r^2}{2h} & -e_{31}r \\ C_{13}^E r & -e_{31}r & (C_{11}^E + C_{12}^E)h \end{pmatrix} \begin{pmatrix} U_z \\ U_r \\ U_R \end{pmatrix}$$

where F_R is the radial force at the outer surface of the tube. Superscript E on the elastic constants has been omitted, as the material is not piezoelectric.

By assuming U_z and U_r to be equal for the two elements, the matrices of (3) and (4) can be assembled into one 4×4 composite matrix with U_z , V , U_r , and U_R as independent degrees of freedom.

The displacement at the interface between rod and tube, U_r , is an internal degree of freedom and is eliminated. The elimination of internal degrees of freedom can be described as follows. First the matrix equation is partitioned into

$$\begin{pmatrix} F_a \\ F_r \end{pmatrix} = \begin{pmatrix} K_{aa} & K_{ar} \\ K_{ra} & K_{rr} \end{pmatrix} \begin{pmatrix} U_a \\ U_r \end{pmatrix} \quad (6)$$

where U_a is a vector of the displacements or electrical potential of the degrees of freedom that are retained. U_r is the displacement that is eliminated. K_{aa} , K_{ar} , and K_{ra} , and K_{rr} are submatrices of the assembled matrix. The external force on the interface between the rod and the tube, F_r is set to zero whereby U_r can be expressed as $U_r = -K_{rr}^{-1} K_{ra} U_a$. This is inserted in the upper part of (6) to give

$$F_a = (K_{aa} - K_{ar} K_{rr}^{-1} K_{ra}) U_a \quad (7)$$

This result is valid for any number of eliminated degrees of freedom, but in the actual case, the result is a 3×3 matrix, a superelement for the composite piezoelectric rod, with exactly the same degrees of freedom as the piezoelectric rod. An equivalent homogeneous rod would therefore have the stiffness matrix already given in (3).

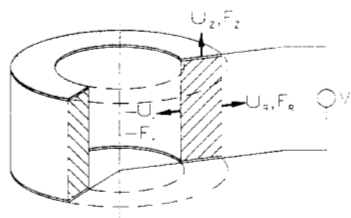


Fig. 1. Tube element.

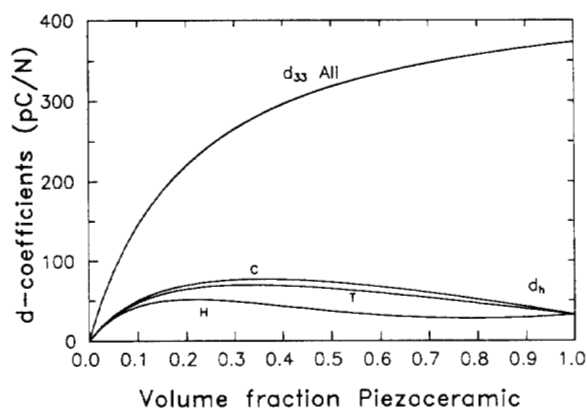
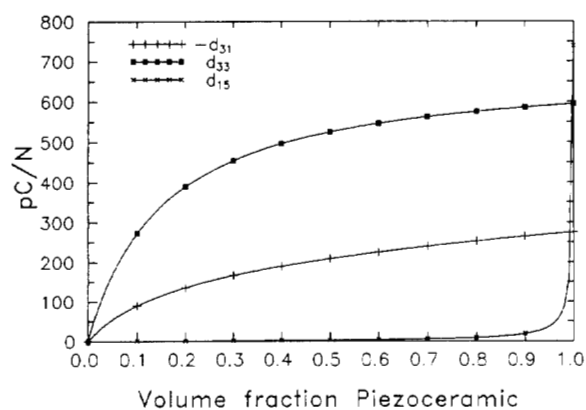
Fig. 2. Comparison of d_{33} and d_h estimates. PZT/Stycast comparison of methods.

Fig. 3. Charge constants for PZT5H/Araldite composite.

The constitutive constants, $(C_{11}^E + C_{12}^E)$, C_{13}^E , C_{33}^E , e_{31} , e_{33} , and ϵ_{33} , can be seen to be directly identifiable in the composite stiffness matrix by the use of (3). The remaining constants, e_{15} and ϵ_{11} , plus C_{11}^E and C_{12}^E individually are found using a cube model. The last three constants may be of little technical interest, but with the complete e -set of constants, the d -, g -, and h -set can be found directly from their definitions by (partial) inversion of the material constant matrices.

III. RESULTS

For comparison the d_{33} and d_h coefficients have been calculated for the PZT5/Stycast example from [8]. The result is shown in Fig. 2. The results are calculated with the method of Haun and Newnham (H) [4], Chan and Unsworth (C) [6], and the concentric tube model (T). The material data used in the calculation can be found in Table I and II. It can be seen that all three methods agree on the

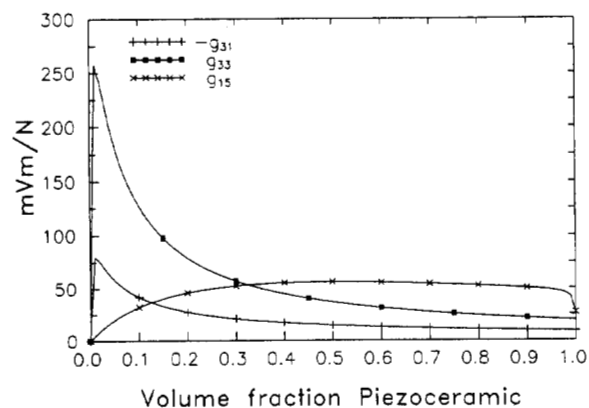


Fig. 4. Voltage constants for PZT5H/Araldite composite.

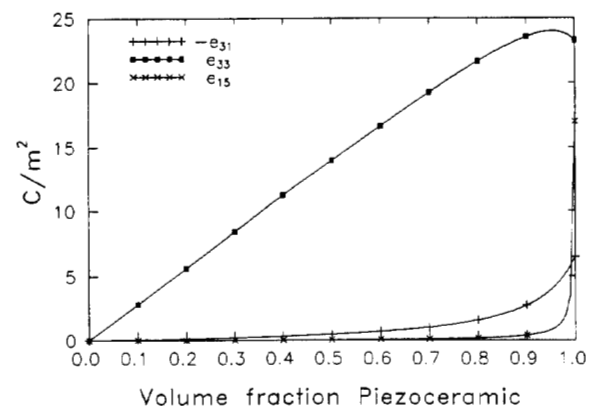
Fig. 5. e -constants for PZT5H/Araldite composite.

TABLE I
MATERIAL DATA FOR PIEZOCERAMICS

| | PZT5 | PZT5H | Units |
|-------------------|------|-------|------------------|
| C_{11}^E | 121 | 126 | GPa |
| C_{12}^E | 75.4 | 79.5 | |
| C_{13}^E | 75.2 | 84.1 | |
| C_{33}^E | 111 | 117 | |
| C_{55}^E | 21.1 | 23.0 | |
| $-e_{31}$ | 5.4 | 6.5 | C/m ² |
| e_{33} | 15.8 | 23.3 | |
| e_{15} | 12.3 | 17.0 | |
| ϵ_{11}^S | 915 | 1695 | * ϵ_0 |
| ϵ_{33}^S | 830 | 1469 | |

TABLE II
MATERIAL DATA FOR POLYMERS

| | Stycast | Araldite | Unit |
|-------|---------|----------|------|
| V_p | 2800 | 2620 | m/s |
| V_s | 1496 | 1400 | |

value of d_{33} whereas there is some difference in the estimates of d_h . The concentric tube model gives results in between the other two methods, but agrees best with the results of Chan and Unsworth. The difference between the methods is so insignificant, that it is not possible from available experimental results to tell which gives the most reliable estimates.

The Figs. 3–6 show the d -, g -, e -, and h -constants for a PZT5H/Araldite composite. It can be noted that these

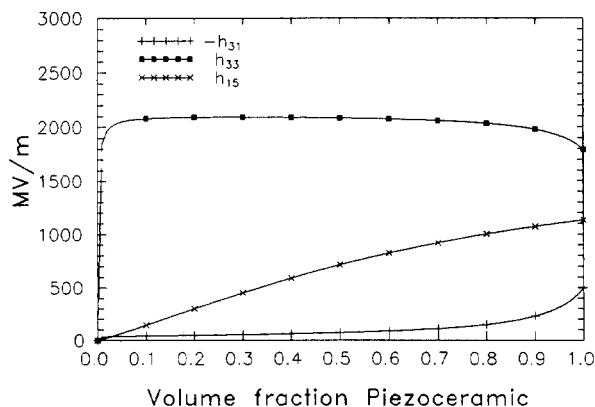


Fig. 6. h -constants for PZT5H/Araldite composite.

constants can not be estimated well with mixture rules of the Voigt or Reuss type. The rapid changes in the shear constants, d_{15} and e_{15} , for large volume fractions of ceramics is partially due to the ideal separation of the rods in the model, and will be less pronounced in real composites, as these will resemble 3-3 composites for large volume fractions. The maximum value of g_{33} occurs at a volume fraction of ceramic as low as 0.0095, which may prove difficult to verify experimentally. The commonly used figure of merit, $d_h g_h$, reaches its maximum at a fraction of ceramics of about 0.07.

IV. CONCLUSION

A new method for estimation of macroscopic characteristics of 1-3 composites has been proposed. The results agree well with those of Chan and Unsworth [6]. From available experimental data it can not be determined which method is most accurate, but the concentric tube model includes an inhomogeneous transverse strain which in theory must be inhomogeneous and does not introduce assumptions, which are more restrictive than the other comparable methods. The concentric tube model is a numerical method, but the numerical calculations are simple and easy to implement.

Furthermore, the presented method can be extended in several directions. The method is recursive, so it is easy to generate a 3-3-3 composite. This specifically covers the case where the rods are coated, but is likely to be valid for most 3-3-3 composites. Numerical refinements can be introduced by using more refined displacement assumptions. The concentric tube model can also be viewed as a constrained or very simplified finite-element model, and by analogy this enables inclusion of other effects, such as first order frequency dependence and damping by use of standard finite element techniques.

The idea of averaging by elimination of internal degrees of freedom followed by matching the reduced matrices for a homogeneous and a composite system is also applicable to the finite element methods as an alternative to using the finite-element method to simulate experimental characterization [10], [11].

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